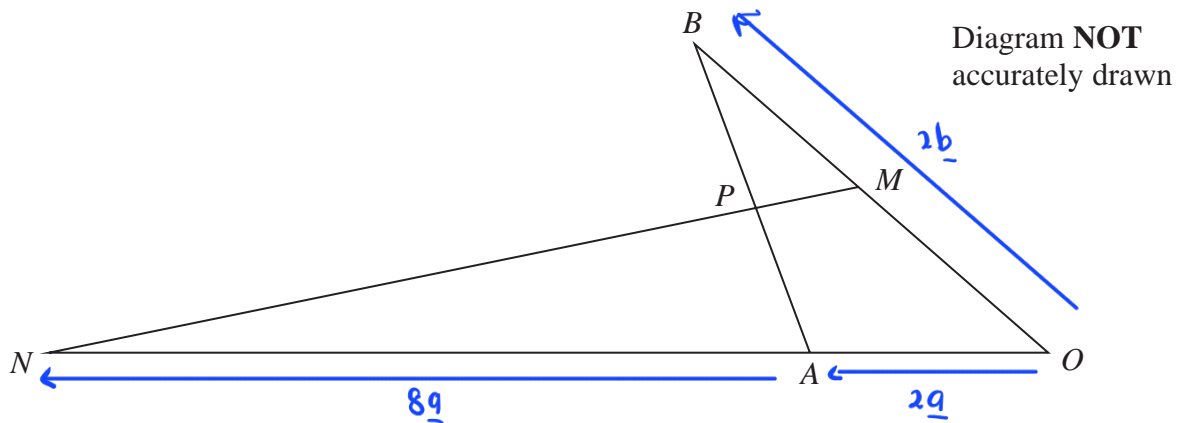


1



OAN , OMB , APB and MPN are straight lines.

$$OA : AN = 1 : 4$$

$$OM : MB = 1 : 1$$

$$\vec{OA} = 2\mathbf{a} \quad \vec{OB} = 2\mathbf{b}$$

By using a **vector method**, find the ratio $AP : PB$
Give your answer in its simplest form.

$$\begin{aligned} \vec{AB} &= \vec{AO} + \vec{OB} \\ &= -2\mathbf{a} + 2\mathbf{b} \quad (1) \end{aligned}$$

$$\begin{aligned} \vec{NM} &= \vec{NO} + \vec{OM} \\ &= -10\mathbf{a} + \mathbf{b} \end{aligned}$$

Let X = fraction of NM

Y = fraction of AB

$$\begin{aligned} \vec{AP} &= \vec{AN} + \vec{NP} \\ &= 8\mathbf{a} + X(-10\mathbf{a} + \mathbf{b}) \quad (1) \end{aligned}$$

$$\vec{AP} = Y(-2\mathbf{a} + 2\mathbf{b}) \quad (1)$$

$$\therefore 8\underline{a} + x(-10\underline{a} + \underline{b}) = y(-2\underline{a} + 2\underline{b})$$

$$8\underline{a} - 10\underline{a}x + \underline{b}x = -2\underline{a}y + 2\underline{b}y$$

$$8\underline{a} = 10\underline{a}x - 2\underline{a}y + 2\underline{b}y - \underline{b}x$$

$$8\underline{a} = \underline{a}(10x - 2y) + \underline{b}(2y - x) \quad (1)$$

$$\underline{b} \text{ term : } 0 = 2y - x$$

$$2y = x \quad (1)$$

$$\underline{a} \text{ term : } 8 = 10x - 2y \quad (2)$$

substitute (1) into (2)

$$8 = 10(2y) - 2y$$

$$8 = 20y - 2y$$

$$y = \frac{8}{18} = \frac{4}{9}$$

since y = fraction of AB,

$$\vec{AP} = \frac{4}{9} \vec{AB}$$

$$AP = \frac{4}{9}, \text{ hence } PB = \frac{5}{9}$$

$$AP : PB = 4 : 5$$

$$4 : 5 \quad (1)$$

(Total for Question 1 is 5 marks)

2 The diagram shows trapezium $OACB$.

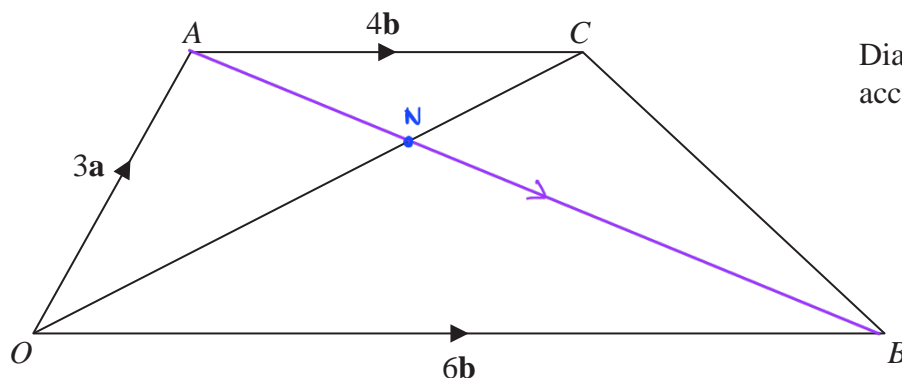


Diagram **NOT**
accurately drawn

$$\vec{OA} = 3\mathbf{a} \quad \vec{OB} = 6\mathbf{b} \quad \vec{AC} = 4\mathbf{b}$$

N is the point on OC such that ANB is a straight line.

Find \vec{ON} as a simplified expression in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{OC} &= \vec{OA} + \vec{AC} \\ &= 3\mathbf{a} + 4\mathbf{b} \quad (1) \end{aligned}$$

$$\begin{aligned} \vec{ON} &= x(\vec{OC}) \\ &= x(3\mathbf{a} + 4\mathbf{b}) \quad (1) \end{aligned}$$

$$\begin{aligned} \vec{AB} &= \vec{AO} + \vec{OB} \\ &= -3\mathbf{a} + 6\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{AN} &= y(\vec{AB}) \\ &= y(-3\mathbf{a} + 6\mathbf{b}) \end{aligned}$$

$$\begin{aligned} \vec{ON} &= \vec{OA} + \vec{AN} \\ &= 3\mathbf{a} + y(-3\mathbf{a} + 6\mathbf{b}) \end{aligned}$$

$$\therefore x(3\mathbf{a} + 4\mathbf{b}) = 3\mathbf{a} + y(-3\mathbf{a} + 6\mathbf{b}) \quad (1)$$

$$\begin{aligned} \mathbf{a} \text{ term : } 3x &= 3 - 3y \\ x &= 1 - y \end{aligned}$$

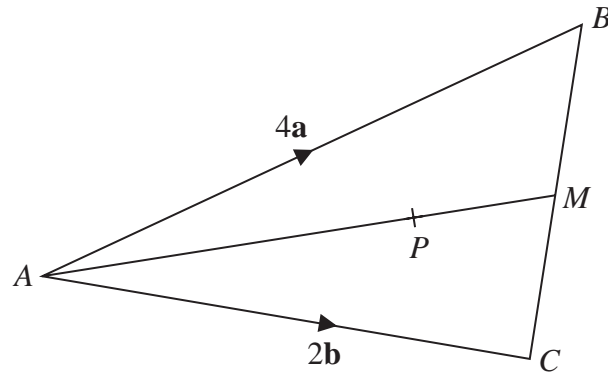
$$\begin{aligned} \vec{ON} &= 0.6(3\mathbf{a} + 4\mathbf{b}) \\ &= 1.8\mathbf{a} + 2.4\mathbf{b} \quad (1) \end{aligned}$$

$$\begin{aligned} \mathbf{b} \text{ term : } 4x &= 6y \\ 4(1-y) &= 6y \\ 4 - 4y &= 6y \\ 4 &= 10y \\ y &= \frac{4}{10} = 0.4, \quad x = 0.6 \quad (1) \end{aligned}$$

$$\vec{ON} = 1.8\mathbf{a} + 2.4\mathbf{b}$$

(Total for Question 2 is 5 marks)

3

Diagram **NOT**
accurately drawn ABC is a triangle.The midpoint of BC is M . P is a point on AM .

$$\vec{AB} = 4\mathbf{a}$$

$$\vec{AC} = 2\mathbf{b}$$

$$\vec{AP} = \frac{3}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$$

$$\begin{aligned}\vec{BC} &= \vec{BA} + \vec{AC} \\ &= -4\mathbf{a} + 2\mathbf{b}\end{aligned}$$

Find the ratio $AP:PM$

$$\begin{aligned}\vec{PM} &= \vec{PA} + \vec{AB} + \vec{BM} \\ &= -\frac{3}{2}\mathbf{a} - \frac{3}{4}\mathbf{b} + 4\mathbf{a} + \frac{1}{2}(-4\mathbf{a} + 2\mathbf{b}) \\ &= -\frac{3}{2}\mathbf{a} + 4\mathbf{a} - 2\mathbf{a} - \frac{3}{4}\mathbf{b} + \mathbf{b} \\ &= \frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b} \quad (1)\end{aligned}$$

$$\begin{aligned}\vec{AP} &= 3\left(\frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b}\right) \\ &= 3\vec{PM} \quad (1)\end{aligned}$$

$$\vec{AM} = \frac{4}{3}\vec{AP}$$

$$\vec{AM} = 4\vec{PM}$$

$$\therefore AP:PM = 3:1 \quad (1)$$

3:1

(Total for Question 3 is 3 marks)

4 $OACB$ is a trapezium.

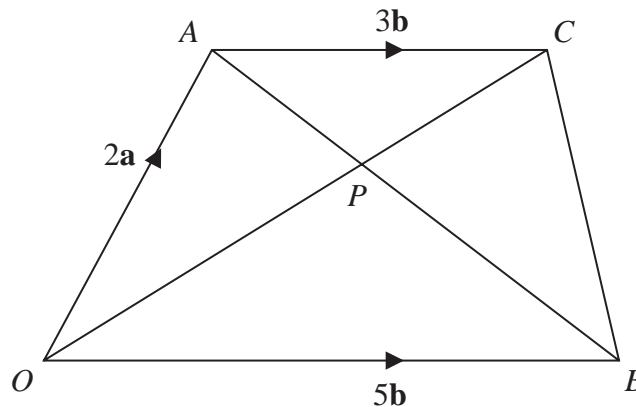


Diagram **NOT**
accurately drawn

$$\vec{OA} = 2\mathbf{a} \quad \vec{OB} = 5\mathbf{b} \quad \vec{AC} = 3\mathbf{b}$$

The diagonals, OC and AB , of the trapezium intersect at the point P .

Find and simplify an expression, in terms of \mathbf{a} and \mathbf{b} , for \vec{OP}
Show your working clearly.

$$\begin{aligned} \vec{OC} &= \vec{OA} + \vec{AC} \\ &= 2\mathbf{a} + 3\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{OP} &= n(\vec{OC}) \\ &= n(2\mathbf{a} + 3\mathbf{b}) \quad \text{①} \end{aligned}$$

$$\begin{aligned} \vec{AB} &= \vec{AO} + \vec{OB} \\ &= -2\mathbf{a} + 5\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{OP} &= \vec{OA} + \vec{AP} \\ &= 2\mathbf{a} + m(\vec{AB}) \\ &= 2\mathbf{a} + m(-2\mathbf{a} + 5\mathbf{b}) \end{aligned}$$

$$n(2\mathbf{a} + 3\mathbf{b}) = 2\mathbf{a} + m(-2\mathbf{a} + 5\mathbf{b}) \quad \text{①}$$

$$2n\mathbf{a} + 3n\mathbf{b} = (2 - 2m)\mathbf{a} + 5m\mathbf{b}$$

$$\begin{aligned} \mathbf{a} : 2n &= 2 - 2m \\ n &= 1 - m \quad \text{--- ①} \end{aligned}$$

$$\mathbf{b} : 3n = 5m \quad \text{--- ②}$$

Substitute ① into ②

$$3(1 - m) = 5m \quad \text{①}$$

$$3 - 3m = 5m$$

$$3 = 8m$$

$$m = \frac{3}{8}, \quad n = 1 - \frac{3}{8} = \frac{5}{8} \quad \text{①}$$

$$\begin{aligned}\vec{OP} &= n(2\vec{a} + 3\vec{b}) \\ &= \frac{5}{8}(2\vec{a} + 3\vec{b}) \\ &= \frac{5}{4}\vec{a} + \frac{15}{8}\vec{b} \quad \textcircled{1}\end{aligned}$$

$$\vec{OP} = \frac{5}{4}\vec{a} + \frac{15}{8}\vec{b}$$

(Total for Question 4 is 5 marks)

5 Here are two vectors.

$$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad \overrightarrow{CB} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

Find, as a column vector, \overrightarrow{AC}

$$\begin{aligned} \overrightarrow{AC} &= \overrightarrow{AB} + \overrightarrow{BC} \\ &= \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} \text{ (1)} \\ &= \begin{pmatrix} 7 \\ -1 \end{pmatrix} \text{ (1)} \end{aligned}$$

$$\begin{pmatrix} 7 \\ -1 \end{pmatrix}$$

(Total for Question 5 is 2 marks)

6 OAB is a triangle.

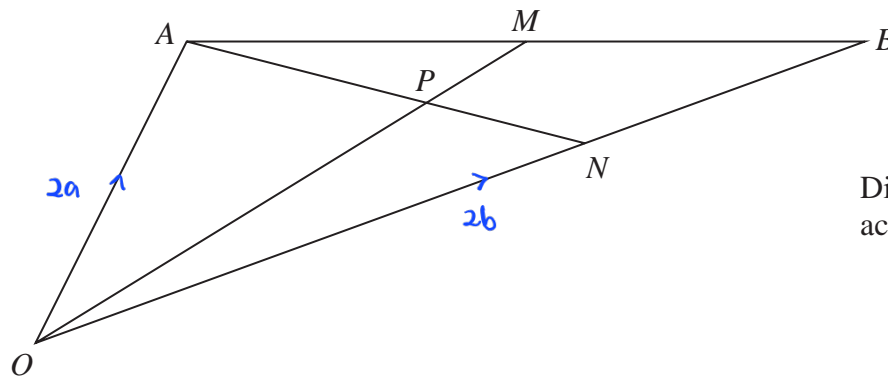


Diagram **NOT**
accurately drawn

$$\vec{OA} = 2\mathbf{a} \text{ and } \vec{OB} = 2\mathbf{b}$$

M is the midpoint of AB .

N is the point on OB such that $ON:NB = 2:1$

P is the point on AN such that OPM is a straight line.

Use a vector method to find $OP:PM$

Show your working clearly.

$$\begin{aligned} \vec{AB} &= \vec{AO} + \vec{OB} \\ &= -2\mathbf{a} + 2\mathbf{b} \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \vec{AM} &= \frac{-2\mathbf{a} + 2\mathbf{b}}{2} \\ &= -\mathbf{a} + \mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{OM} &= \vec{OA} + \vec{AM} \\ &= 2\mathbf{a} + (-\mathbf{a} + \mathbf{b}) \\ &= \mathbf{a} + \mathbf{b} \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \vec{OP} &= n(\vec{OM}) \\ &= n(\mathbf{a} + \mathbf{b}) \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \vec{ON} &= \frac{2}{3} 2\mathbf{b} \\ &= \frac{4}{3} \mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{AN} &= \vec{AO} + \vec{ON} \\ &= -2\mathbf{a} + \frac{4}{3} \mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{AP} &= m(\vec{AN}) \\ &= m(-2\mathbf{a} + \frac{4}{3} \mathbf{b}) \end{aligned}$$

$$\vec{AP} = \vec{AO} + \vec{OP}$$

$$m(-2\mathbf{a} + \frac{4}{3} \mathbf{b}) = -2\mathbf{a} + n(\mathbf{a} + \mathbf{b}) \quad \textcircled{1}$$

$$\mathbf{a} : -2m = -2 + n \quad \textcircled{2}$$

$$\mathbf{b} : \frac{4}{3}m = n \quad \textcircled{1}$$

subs ① into ②

$$-2m = -2 + \frac{4}{3}m$$

$$2 = \frac{4}{3}m + 2m$$

$$2 = \frac{10}{3}m$$

$$m = \frac{3}{5}$$

$$n = \frac{4}{5} \quad \text{①}$$

$$\vec{OP} = \frac{4}{5}(a+b)$$

$$\vec{OP} = \frac{4}{5}(\vec{OM})$$

$$\vec{PM} = \frac{1}{5}(\vec{OM})$$

$$\vec{OP} : \vec{PM} = 4 : 1 \quad \text{①}$$

4 : 1

(Total for Question 6 is 6 marks)

7 $ABCD$ is a parallelogram and ADM is a straight line.

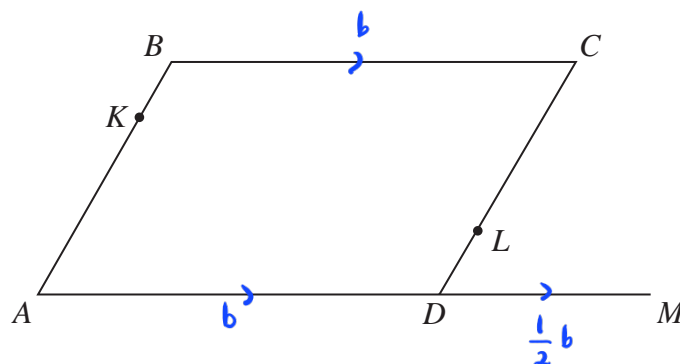


Diagram **NOT**
accurately drawn

$$\vec{AB} = \mathbf{a} \quad \vec{BC} = \mathbf{b} \quad \vec{DM} = \frac{1}{2}\mathbf{b}$$

K is the point on AB such that $AK:AB = \lambda:1$

L is the point on CD such that $CL:CD = \mu:1$

KLM is a straight line.

Given that $\lambda:\mu = 1:2$

use a vector method to find the value of λ and the value of μ

$$\vec{AK} = \lambda \underline{a}, \quad \vec{KB} = (1-\lambda)\underline{a}, \quad \vec{CL} = -\mu \underline{a}, \quad \vec{DL} = (1-\mu)\underline{a} \quad \textcircled{1}$$

$$\begin{aligned} \vec{KL} &= \vec{KA} + \vec{AD} + \vec{DL} \\ &= -\lambda \underline{a} + \underline{b} + (1-\mu)\underline{a} \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} \vec{LM} &= \vec{LD} + \vec{DM} \\ &= -(1-\mu)\underline{a} + \frac{1}{2}\underline{b} = -(1-2\lambda)\underline{a} + \frac{1}{2}\underline{b} \quad \textcircled{1} \end{aligned}$$

$\mu = 2\lambda$

$$\begin{aligned} \vec{KM} &= \vec{KA} + \vec{AD} + \vec{DM} \\ &= -\lambda \underline{a} + \underline{b} + \frac{1}{2}\underline{b} \end{aligned}$$

$$\vec{LM} = x \vec{KM}$$

$$\underline{a} : 2\lambda - 1 = -\lambda x \quad \textcircled{1}$$

$$\underline{b} : \frac{1}{2} = \frac{3}{2}x$$

$$x = \frac{1}{3} \quad \textcircled{2}$$

substitute $\textcircled{2}$ into $\textcircled{1}$:

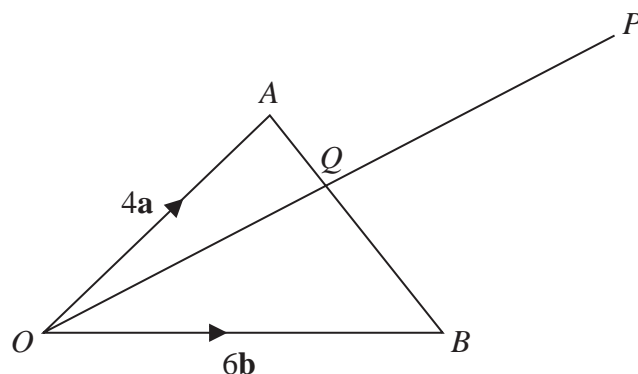
$$2\lambda - 1 = -\frac{1}{3}\lambda, \quad \lambda = \frac{3}{7} \quad \textcircled{1}, \quad \mu = 2 \times \frac{3}{7} = \frac{6}{7} \quad \textcircled{1}$$

$$\lambda = \frac{3}{7}$$

$$\mu = \frac{6}{7}$$

(Total for Question 7 is 5 marks)

8

Diagram NOT
accurately drawn OAB is a triangle. Q is the point on AB such that OQP is a straight line.

$$\vec{OA} = 4\mathbf{a} \quad \vec{OB} = 6\mathbf{b} \quad \vec{AP} = 2\mathbf{a} + 8\mathbf{b}$$

Using a vector method, find the ratio $AQ:QB$

$$\begin{aligned} \vec{OP} &= \vec{OA} + \vec{AP} \\ &= 4\mathbf{a} + 2\mathbf{a} + 8\mathbf{b} \quad (1) \\ &= 6\mathbf{a} + 8\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{AB} &= \vec{AO} + \vec{OB} \\ &= -4\mathbf{a} + 6\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{AQ} &= \lambda \vec{AB} \\ &= \lambda(-4\mathbf{a} + 6\mathbf{b}) \quad (1) \\ &= -4\lambda\mathbf{a} + 6\lambda\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{AQ} &= \vec{AO} + \vec{OQ} \\ &= -4\mathbf{a} + \mu \vec{OP} \\ &= -4\mathbf{a} + \mu(6\mathbf{a} + 8\mathbf{b}) \quad (1) \\ &= (-4 + 6\mu)\mathbf{a} + 8\mu\mathbf{b} \end{aligned}$$

$$\underline{a} \text{ terms: } -4\lambda = -4 + 6\mu$$

$$\underline{b} \text{ terms: } 6\lambda = 8\mu$$

$$\mu = \frac{3}{4}\lambda$$

$$-4\lambda = -4 + 6\left(\frac{3}{4}\lambda\right)$$

$$-4\lambda = -4 + \frac{9}{2}\lambda$$

$$\lambda = \frac{8}{17} \quad (1)$$

$$AQ:QB$$

$$\lambda : 1 - \lambda$$

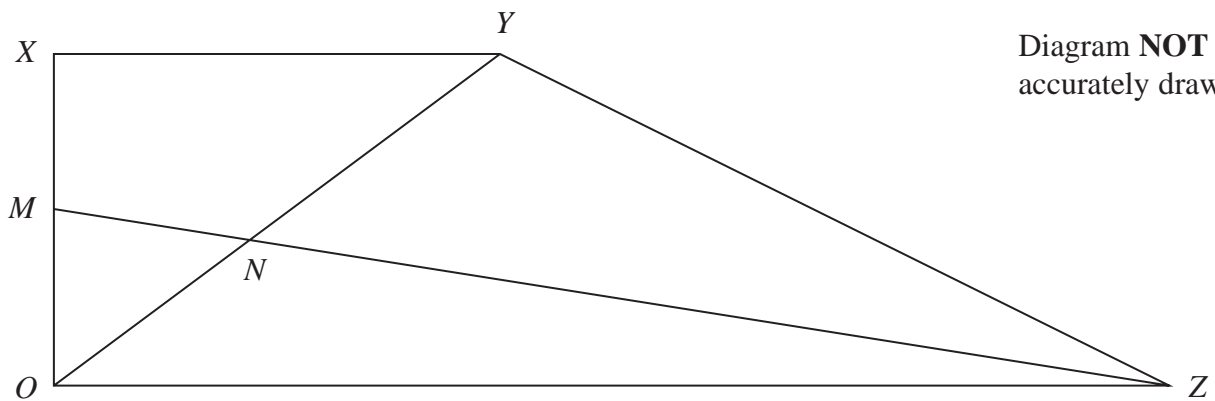
$$\frac{8}{17} : \frac{9}{17}$$

$$8:9 \quad (1)$$

$$AQ:QB = \dots\dots\dots$$

(Total for Question 8 is 5 marks)

9 $OXYZ$ is a trapezium.



$$\vec{OX} = \mathbf{a}$$

$$\vec{XY} = \mathbf{b}$$

$$\vec{OZ} = 3\mathbf{b}$$

M is the midpoint of OX

N is the point such that MNZ and ONY are straight lines.

Given that $ON : OY = \lambda : 1$

use a vector method to find the value of λ

$$\begin{aligned}\vec{ON} &= \lambda(\vec{OY}) \quad (1) \\ &= \lambda(\vec{OX} + \vec{XY}) \\ &= \lambda(\underline{\mathbf{a}} + \underline{\mathbf{b}})\end{aligned}$$

$$\begin{aligned}\vec{MN} &= \vec{MO} + \vec{ON} \quad (1) \\ &= -0.5\underline{\mathbf{a}} + \lambda\underline{\mathbf{a}} + \lambda\underline{\mathbf{b}}\end{aligned}$$

$$\begin{aligned}\vec{MN} &= \mu(\vec{MZ}) \quad (1) \\ &= \mu(\vec{MO} + \vec{OZ}) \\ &= \mu(-0.5\underline{\mathbf{a}} + 3\underline{\mathbf{b}})\end{aligned}$$

$$(\lambda - 0.5)\underline{\mathbf{a}} + \lambda\underline{\mathbf{b}} = (-0.5\mu)\underline{\mathbf{a}} + (3\mu)\underline{\mathbf{b}}$$

$$\underline{\mathbf{a}} \text{ term: } \lambda - 0.5 = -0.5\mu \quad (1)$$

$$\underline{\mathbf{b}} \text{ term: } \lambda = 3\mu \quad (2)$$

Substitute ② into ① :

$$3\lambda - 0.5 = -0.5\lambda$$

$$3.5\lambda = 0.5$$

$$\lambda = \frac{1}{7}$$

$$\lambda = 3\left(\frac{1}{7}\right)$$

$$= \frac{3}{7} \text{ ①}$$

$$\lambda = \frac{3}{7}$$

(Total for Question 9 is 5 marks)

10 The diagram shows triangle OAB

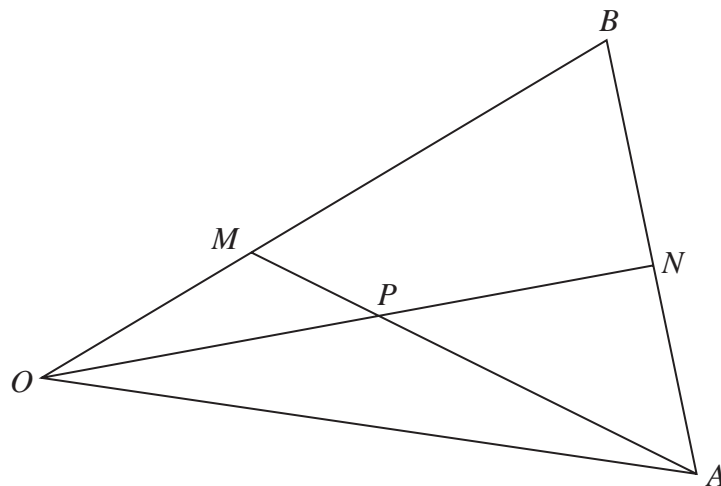


Diagram **NOT**
accurately drawn

$$\vec{OA} = 8\mathbf{a} \quad \vec{OB} = 6\mathbf{b}$$

M is the point on OB such that $OM:MB = 1:2$

N is the midpoint of AB

P is the point of intersection of ON and AM

Using a vector method, find \vec{OP} as a simplified expression in terms of \mathbf{a} and \mathbf{b}
Show your working clearly.

$$\begin{aligned} \vec{BA} &= \vec{BO} + \vec{OA} & \vec{BN} &= \frac{1}{2}(-6\mathbf{b} + 8\mathbf{a}) \\ &= -6\mathbf{b} + 8\mathbf{a} & &= -3\mathbf{b} + 4\mathbf{a} \end{aligned}$$

$$\begin{aligned} \vec{ON} &= \vec{OB} + \vec{BN} \\ &= 6\mathbf{b} + (-3\mathbf{b} + 4\mathbf{a}) \\ &= 4\mathbf{a} + 3\mathbf{b} \quad \textcircled{1} \end{aligned}$$

$$\vec{OP} = \lambda(4\mathbf{a} + 3\mathbf{b}) \quad \textcircled{1}$$

$$\begin{aligned} \vec{OP} &= \vec{OA} + \vec{AP} \\ &= 8\mathbf{a} + x\vec{AM} \\ &= 8\mathbf{a} + x(\vec{AB} + \vec{BM}) \\ &= 8\mathbf{a} + x(6\mathbf{b} - 8\mathbf{a} - 4\mathbf{b}) \\ &= 8\mathbf{a} + x(2\mathbf{b} - 8\mathbf{a}) \quad \textcircled{1} \end{aligned}$$

$$\underline{a} \text{ term: } 4h = 8 - 8x \quad \text{--- ①}$$

$$\underline{b} \text{ term: } 3h = 2x$$

$$h = \frac{2}{3}x \quad \text{--- ②}$$

subs ② into ① :

$$4\left(\frac{2}{3}x\right) = 8 - 8x$$

$$\frac{8}{3}x = 8 - 8x$$

$$8x = 24 - 24x$$

$$32x = 24$$

$$x = \frac{24}{32} = \frac{3}{4}$$

$$h = \frac{2}{3}\left(\frac{3}{4}\right) \quad \text{①}$$

$$= \frac{1}{2}$$

$$\vec{OP} = \frac{1}{2} (4\underline{a} + 3\underline{b})$$

$$= 2\underline{a} + \frac{3}{2}\underline{b} \quad \text{①}$$

$$\vec{OP} = \underline{2a} + \frac{3}{2}\underline{b}$$

(Total for Question 10 is 5 marks)

11 The diagram shows triangle OAB with OA extended to E

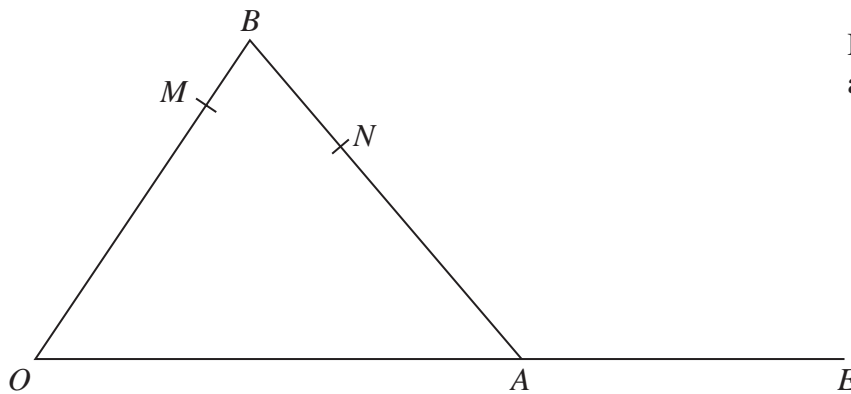


Diagram **NOT**
accurately drawn

$$\vec{OA} = \mathbf{a} \quad \vec{OB} = \mathbf{b}$$

M is the point on OB such that $OM:MB = 4:1$

N is the point on AB such that $AN:NB = 3:2$

$OA:AE = 5:3$

- (a) Find an expression for \vec{ON} in terms of \mathbf{a} and \mathbf{b}
Give your answer in its simplest form.

$$\begin{aligned} \vec{ON} &= \vec{OB} + \vec{BN} \\ &= \underline{\underline{\mathbf{b}}} + \frac{2}{5} \vec{BA} \\ &= \underline{\underline{\mathbf{b}}} + \frac{2}{5} (\vec{BO} + \vec{OA}) \\ &= \underline{\underline{\mathbf{b}}} + \frac{2}{5} (-\underline{\underline{\mathbf{b}}} + \underline{\underline{\mathbf{a}}}) \\ &= \frac{2}{5} \underline{\underline{\mathbf{a}}} + \frac{3}{5} \underline{\underline{\mathbf{b}}} \end{aligned} \quad (1)$$

$$\vec{ON} = \frac{2}{5} \underline{\underline{\mathbf{a}}} + \frac{3}{5} \underline{\underline{\mathbf{b}}} \quad (2)$$

(Total for Question 11 is 2 marks)

12 Here are two vectors.

$$\vec{BA} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} \quad \vec{BC} = \begin{pmatrix} 9 \\ 1 \end{pmatrix}$$

Find \vec{AC} as a column vector.

$$\begin{aligned} \vec{AC} &= \vec{AB} + \vec{BC} \\ &= \begin{bmatrix} 5 \\ -4 \end{bmatrix} + \begin{bmatrix} 9 \\ 1 \end{bmatrix} \quad (1) \\ &= \begin{bmatrix} 14 \\ -3 \end{bmatrix} \quad (1) \end{aligned}$$

$$\vec{AC} = \begin{pmatrix} 14 \\ -3 \end{pmatrix}$$

(Total for Question 12 is 2 marks)

13 $OAED$ is a quadrilateral.

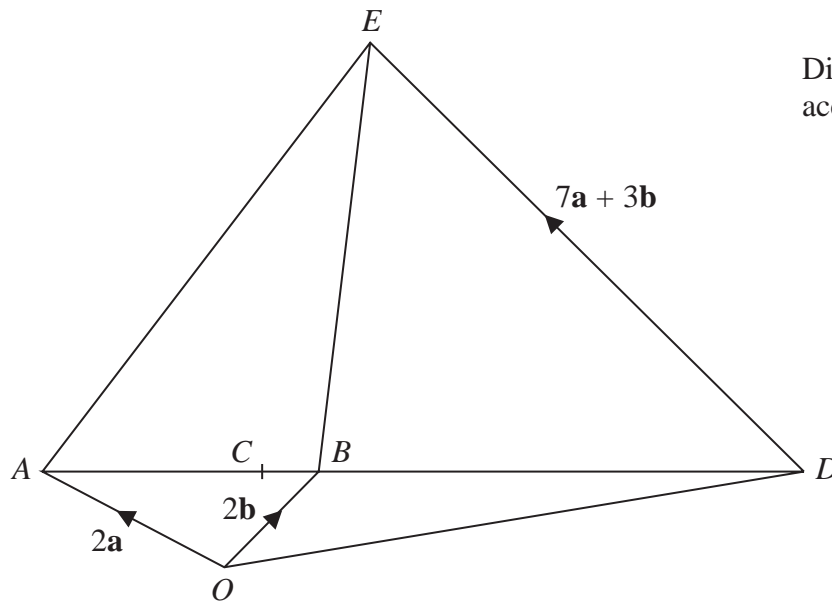


Diagram **NOT**
accurately drawn

$$\vec{OA} = 2\mathbf{a} \quad \vec{OB} = 2\mathbf{b} \quad \vec{DE} = 7\mathbf{a} + 3\mathbf{b}$$

$$AB:BD = 1:2$$

The point C on AB is such that OCE is a straight line.

Use a vector method to find the ratio of $OC:CE$

$$\begin{aligned} \vec{AB} &= \vec{AO} + \vec{OB} & \vec{AD} &= \vec{AB} + \vec{BD} \\ &= -2\mathbf{a} + 2\mathbf{b} & &= -2\mathbf{a} + 2\mathbf{b} + 2(-2\mathbf{a} + 2\mathbf{b}) \\ & & &= -2\mathbf{a} - 4\mathbf{a} + 2\mathbf{b} + 4\mathbf{b} \\ & & &= -6\mathbf{a} + 6\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{OE} &= \vec{OA} + \vec{AD} + \vec{DE} \\ &= 2\mathbf{a} + (-6\mathbf{a} + 6\mathbf{b}) + 7\mathbf{a} + 3\mathbf{b} \\ &= 2\mathbf{a} - 6\mathbf{a} + 7\mathbf{a} + 6\mathbf{b} + 3\mathbf{b} \\ &= 3\mathbf{a} + 9\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{OC} &= \vec{OA} + \vec{AC} \\ &= 2\mathbf{a} + \lambda(-2\mathbf{a} + 2\mathbf{b}) \\ &= (2-2\lambda)\mathbf{a} + 2\lambda\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{OC} &= \mu(\vec{OE}) \\ &= \mu(3\mathbf{a} + 9\mathbf{b}) \end{aligned}$$

$$\underline{a} : 2 - 2h = 3u - \textcircled{1}$$

$$\underline{b} : 2h = 9u$$

$$h = \frac{9}{2}u - \textcircled{2} \quad \textcircled{1}$$

$$2 - 2\left(\frac{9}{2}u\right) = 3u$$

$$2 = 12u$$

$$u = \frac{1}{6}$$

$$h = \frac{9}{2}\left(\frac{1}{6}\right)$$

$$h = \frac{3}{4}$$

$$OC : CE = 1 : 5$$

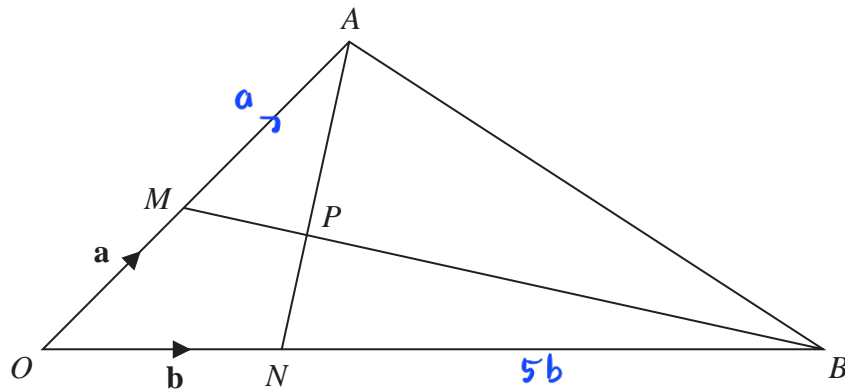
$$1 : 5 \quad \textcircled{1}$$

$$\vec{OC} = \frac{1}{6}(\vec{OE})$$

$$\vec{CE} = \frac{5}{6}(\vec{OE})$$

(Total for Question 13 is 5 marks)

14

Diagram **NOT**
accurately drawn

OMA , ONB , MPB and NPA are straight lines.

M is the midpoint of OA

$ON:NB = 1:5$

$$\vec{OM} = \mathbf{a} \quad \vec{ON} = \mathbf{b}$$

(a) Find in terms of \mathbf{a} and \mathbf{b} the vector \vec{AN}

$$\begin{aligned} \vec{AN} &= \vec{AO} + \vec{ON} \\ &= -2\mathbf{a} + \mathbf{b} \end{aligned}$$

$$-2\mathbf{a} + \mathbf{b} \quad (1)$$

(1)

(b) Use a vector method to find the ratio $AP:PN$

$$\begin{aligned} \vec{OP} &= \vec{OM} + \vec{MP} & \vec{OP} &= \vec{OA} + \vec{AP} \\ &= \mathbf{a} + \lambda(\vec{MB}) & &= 2\mathbf{a} + \mu(\vec{AN}) \\ &= \mathbf{a} + \lambda(-\mathbf{a} + 6\mathbf{b}) & &= 2\mathbf{a} + \mu(-2\mathbf{a} + \mathbf{b}) \end{aligned} \quad (1)$$

$$\mathbf{a} : 1 - \lambda = 2 - 2\mu \quad (1)$$

$$\mathbf{b} : 6\lambda = \mu \quad (2) \quad (1)$$

substitute (2) into (1) :

$$1 - \lambda = 2 - 2(6\lambda)$$

$$1 - \lambda = 2 - 12\lambda$$

$$11\lambda = 1$$

$$\lambda = \frac{1}{11}$$

$$\mu = \frac{6}{11}$$

$$\vec{AP} = \frac{6}{11} \vec{AN}$$

$$\vec{AP} : \vec{PN} = 6 : 5$$

①

$$AP : PN = \frac{6}{5}$$

(4)

(Total for Question 14 is 5 marks)